Olber's Paradox Solved

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1 Introduction

Why is the night sky dark? According to Olber's paradox the night (and day) sky should be infinitely bright. In this article it is considerd how Olber comes to this remarkable conclusion, and a solution to solve his paradox is proposed. This is a solution which does *not* require rejecting any of Olber's basic assumptions of the universe.

Olber's paradox arises when we make the following assumptions about the universe:

1. it is an infinite Euclidean space;
2. it has infinite age;
3. matter is, on average, uniformly distributed through the entire universe;
4. it is static.

We call this a Newtonian Universe.

2 Definitions

2.1 Universal average out-bound energy flow

Consider a small volume element $\mathrm{d}V$ at position $\vec{r}$ in the universe. Let $p_{out}(\vec{r})\mathrm{d}V$ be the amount of energy per time unit streaming out of this element. Similar, let $p_{in}(\vec{r})\mathrm{d}V$ be the in-bound energy flow for this element. The net out-bound energy $p_{no}\mathrm{d}V$ flow for this element is thus given by

$$p_{no}\mathrm{d}V = (p_{out} - p_{in})\mathrm{d}V \quad (1)$$

Assumption 3 implies that we can define a universal average net out-bound energy flow p as follows

$$p \equiv \lim_{V\to\infty} \frac{1}{V}\int_V (p_{out} - p_{in})\mathrm{d}V \quad (2)$$

We can deduce several properties for p. From assumption 4 we conclude

$$\frac{\mathrm{d}}{\mathrm{d}t}p = 0 \quad (3)$$

Furthermore it is likely that p is proportional to the universal average mass density ρ, which we can define in a similar way. With $\rho_{universe}(\vec{r})$ being the actual mass density at position $\vec{r}$, this leads to the following two equations

$$\rho \equiv \lim_{V\to\infty} \frac{1}{V}\int_V \rho_{universe}\mathrm{d}V \tag{4}$$

$$p = k_p\rho \tag{5}$$

With k_p a universal constant *defined* by (5).

2.2 Universal average attenuation length

Consider particles flying through space. Let $P_{na}(a+b)$ be the probability of a particle *not* being absorbed (or scattered or decayed) flying a distance $a+b$. Let $P_{na}(a)$ and $P_{na}(b)$ be the probability of that particle *not* being absorbed flying the distances a and b of distance $a+b$. Since $P_{na}(a)$ and $P_{na}(b)$ are independant of each other, the following is true

$$P_{na}(a+b) = P_{na}(a)P_{na}(b) \tag{6}$$

Furthermore the following is trivial

$$P_{na}(0) = 0 \tag{7}$$

Assuming space has, on average, uniform attenuation properties (follows from assumption 3), equations (6) and (7) lead to

$$P_{na}(r) = e^{-r/\lambda} \tag{8}$$

With $P_{na}(r)$ the probability of a particle *not* being absorbed flying a distance r. This means a beam of particles, and thus a beam of energy (which consists of particles too), attenuates over a distance r with a factor $e^{-r/\lambda}$.

Let $\lambda_{universe}(\vec{r})$ be the actual value of the attenution length at position $\vec{r}$. Similar to the in-bound energy flow and mass density we can define a universal average attenuation length λ as

$$\lambda \equiv \lim_{V\to\infty} \frac{1}{V}\int_V \lambda_{universe}\mathrm{d}V \tag{9}$$

Again it is likey that λ is proportional to ρ, thus

$$\lambda = k_\lambda\rho \tag{10}$$

With k_λ a universal constant *defined* by (10). Finally, assumption 4 leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}\lambda = 0 \tag{11}$$

3 The Paradox

Consider the amount of energy per time unit $\mathrm{d}P$ reaching the Earth from volume element $\mathrm{d}V$ at distance $\vec{r}$ from the center of the Earth. This value is given by

$$dP = p\,\mathrm{d}V\frac{\pi R^2}{4\pi r^2} \tag{12}$$

With R the radius of the Earth. Thus the total amount of energy reaching the Earth per time unit P is

$$\begin{aligned} P &= \lim_{V\to\infty} \int_V p \frac{\pi R^2}{4\pi r^2} \mathrm{d}V & (13) \\ &= \int_0^\infty p \frac{\pi R^2}{4\pi r^2} 4\pi r^2 \mathrm{d}r & (14) \\ &= \pi R^2 p \int_0^\infty \mathrm{d}r & (15) \\ &= \infty & (16) \end{aligned}$$

An *infinite* number. This means that the temperature on Earth should be infinite and the sky should be infinitely bright. Clearly this is not true. This is Olber's paradox.

4 The Solution

In the previuos calculation of the amount of energy per time unit reaching us, shielding of outer spherical shells by inner shells has not been taken into account. If we do take this into account, this results in a modification of (12) to

$$dP = p\,\mathrm{d}V \frac{\pi R^2}{4\pi r^2} e^{-r/\lambda} \quad (17)$$

The energy reaching us from the distant volume element is attenuated with a factor $e^{-r/\lambda}$. The total amount of energy reaching the Earth per time unit P is now given by

$$\begin{aligned} P &= \lim_{V\to\infty} \int_V p \frac{\pi R^2}{4\pi r^2} e^{-r/\lambda} \mathrm{d}V & (18) \\ &= \int_0^\infty p \frac{\pi R^2}{4\pi r^2} e^{-r/\lambda} 4\pi r^2 \mathrm{d}r & (19) \\ &= \pi R^2 p \int_0^\infty e^{-r/\lambda} \mathrm{d}r & (20) \\ &= \pi R^2 p \lambda & (21) \end{aligned}$$

A *finite* number. The paradox has been solved!

5 Further Analysis

The energy flux Φ at the Earth's surface follows from (21)

$$\begin{aligned} \Phi &= \frac{P}{4\pi R^2} & (22) \\ &= \frac{1}{4} p \lambda & (23) \end{aligned}$$

Substituting (5) and (10) gives

$$\Phi = \frac{1}{4} k_p k_\lambda \rho^2 \quad (24)$$

The quantities Φ, k_p and k_λ are measurable quantities. No doubt Φ can be easier determined than k_p and k_λ. But should we find a way to determine these quantities, we have (using (24)) a way of determining the average mass density ρ the entire universe! That is, the average mass density of visible mass, since this equation is concerned with mass that interacts with radiation. Dark matter (if it exists), which by definition doesn't interact with radiation except by gravitation, is excluded from this deduction.

6 Gravity Paradox

Similar to Olber's paradox one can consider the gravitional potential energy ϕ a mass will experience at any point in space. The contribution from a volume element $\mathrm{d}V$ at distance $\vec{r}$ from an arbitrarily chosen point is given by

$$\mathrm{d}\phi = -\frac{G\rho\mathrm{d}V}{r} \tag{25}$$

Notice: this time ρ includes visible and non-visible mass (dark matter). Thus the total ϕ at any point chosen is

$$\begin{aligned} \phi &= \lim_{V\to\infty}\int_V \frac{-G\rho\mathrm{d}V}{r} &(26)\\ &= \int_0^\infty \frac{-G\rho 4\pi r^2\mathrm{d}r}{r} &(27)\\ &= -G\rho 4\pi\int_0^\infty r\mathrm{d}r &(28)\\ &= -\infty^2 &(29)\end{aligned}$$

Infinity to the power of 2! An even worse result than Olber's paradox. And since gravitation can not be shielded, another explanation is needed for this phenomenon.

The solution is the assumption matter is on average uniformy distributed through space. That results in the cancelation of gravitional forces, since for every element $\mathrm{d}V$ there is one exact opposit element exercising an exact opposite force. Equivalently, one can never observe this potential energy, since it is infinite but equal at any point in space. Hence, there is no gradient and thus no resulting force. And since it is unobservable, it is not relevant. One can consider it to be zero. A sort of second normalization (the first one being the statement that ϕ as a result from a mass m equals zero at infinite distance from it).

An odd but maybe satifying solution. However, less convenient than the one for Olber's paradox. Maybe this strange inifinite potential has something to do with Mach's principle? Maybe a more neat nomalization will not race to infinity but will have some remaining effect explaining Mach's principle?

7 Conclusion

There is a way of solving Olber's paradox without resorting to rejecting one or more of Olber's basic assumptions of the universe. For example by stating the universe is finite, has a finite age, etc. A more conventional solution, leaving the assumptions intact, exists as well. This conventinal solution doesn't require rejecting any of the assumptions, but it doesn't forbid doing so either.

Unfortunately there is no such solution for the Gravity paradox. There is another solution however, but much less satisfying.